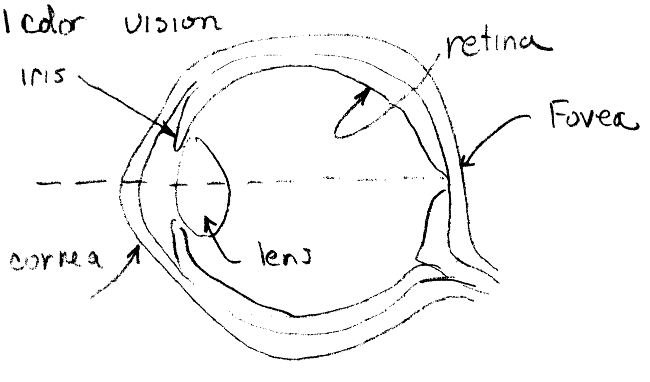
Castleman, Digital Image Processing
Ch. 21 Color & Multi-spectral Image Processing

multispectral image gray level $= f(x, y, \lambda)$

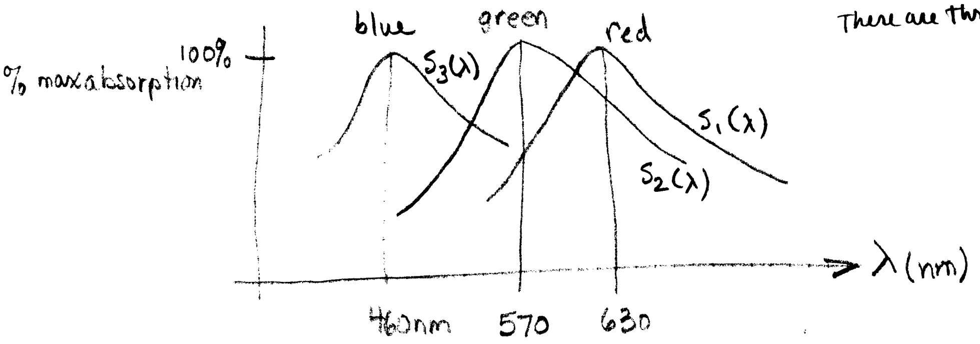
color image processing when λ restricted to the red, green and blue bands to which the eye is sensitive

21.3.1 color vision



photoreceptors
- rods – monochromatic, very light sensitive
- cones – not nearly as sensitive, but respond to red, green and blue.

There are three types



Fundamentally based upon Theory of Thomas Young (1802) that you can reproduce any color by mixing 3 primary colors.

21.3.2 Tricolor Imaging

since the eye is tricolor almost all developments (digitizers, displays, printers, displays, TV cameras) have been devoted to tri-color

best examples – television – uses red, green & blue filters and three separate sensors.

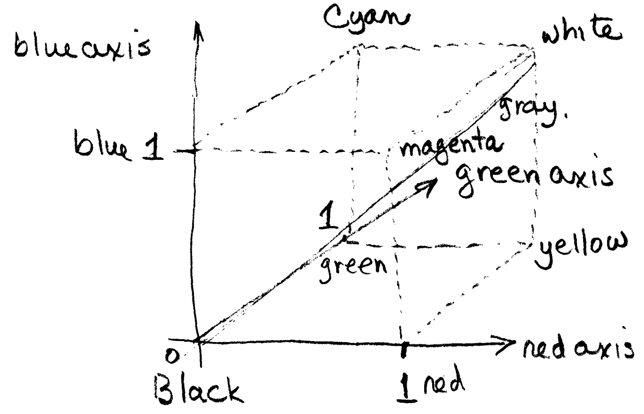
color photography – three sandwiched emulsions

color displays – superimpose red, green & blue images

Best to visualize color as an image having three gray levels corresponding to red, green and blue.

21.3.3 Color Specification

RGB format (most common) (similar to NTSC receiver).



the diagonal of this cube is the line of all gray values.

red, green, blue – primary colors

yellow, cyan (blue-green) and magenta (purple) – secondary colors

a histogram of a RGB image is actually a scatter plot of points in space

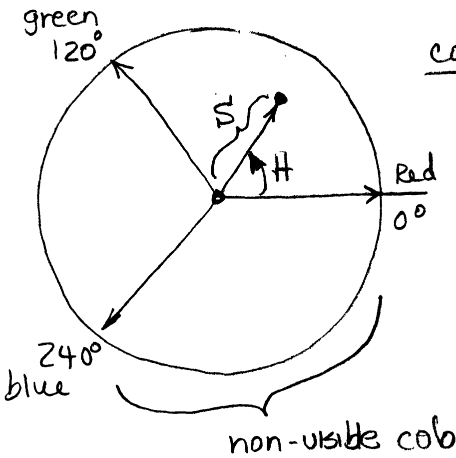
HSI (based upon way humans see color)

I = intensity = average of R, G, B values.

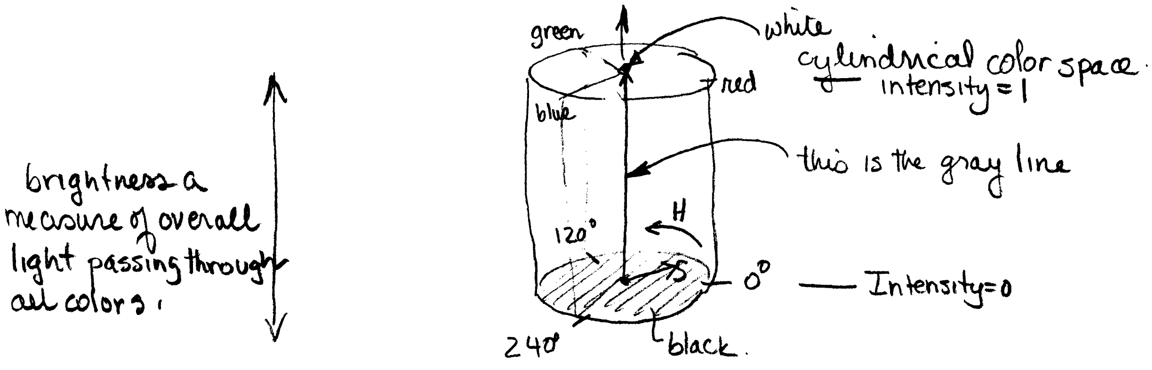
this is one way to convert color to 'intensity

H = hue, determined by dominant wavelength

S = saturation, magnitude of hue relative to other wavelengths

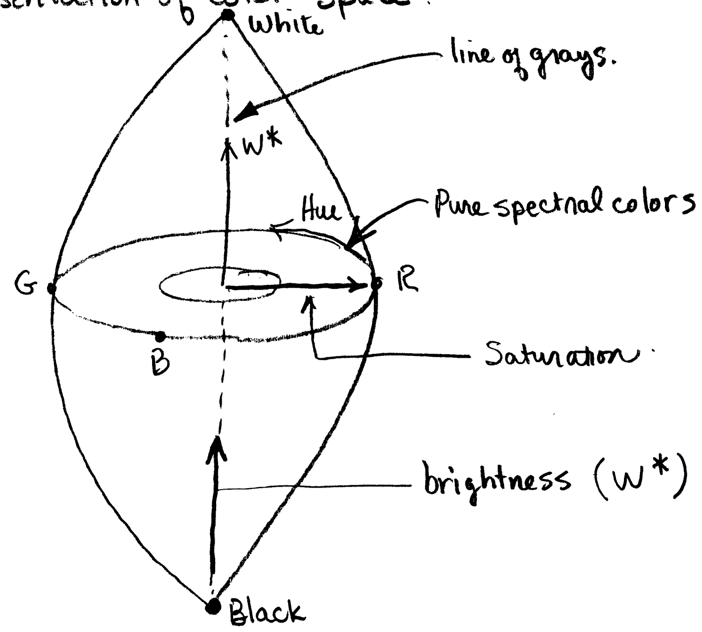


color circle



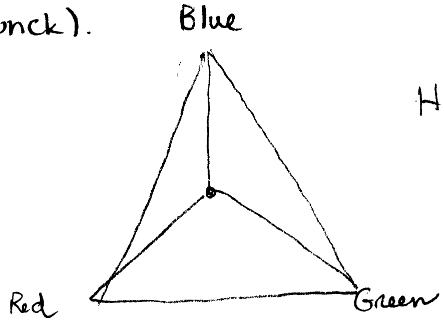
3.7 (Jain) Color Representation

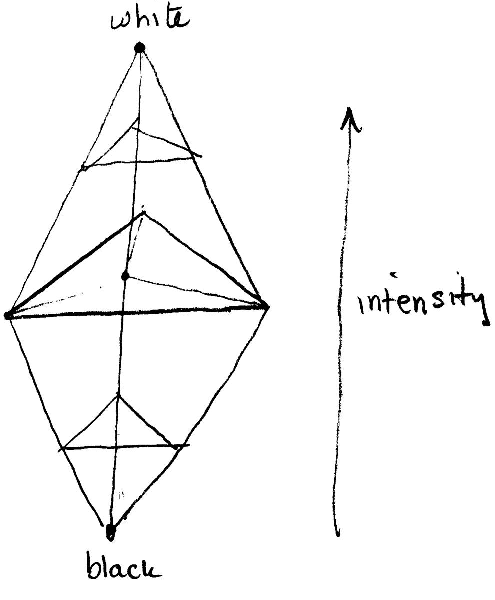
Perceptual Representation of color space.



10.4 (Jain, Kasturi, Schunck).

HSI color triangle





21.3.4.1 RGB → HSI Conversion

first step - rotate the RGB cube

without the details we can rotate the cube diagonal to become the axis of the cylinder (both are the gray line)

the rotated spce is given by

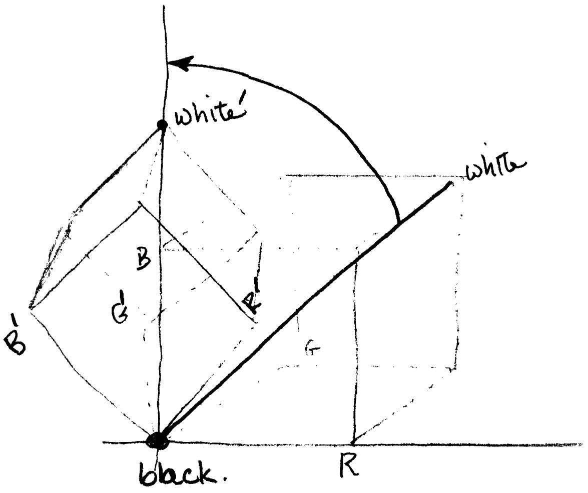
not to be confused with CIE x, y, z.

$$\left\{\begin{aligned} X &= \frac{1}{\sqrt{6}}\left[2R - G - B\right] \\ y &= \frac{1}{\sqrt{2}}\left[G - B\right] \\ z &= \frac{1}{\sqrt{3}}\left[R + G + B\right] \quad \text{this is the intensity} \end{aligned}\right\} \text{ this is just rotation}$$

now convert to polar

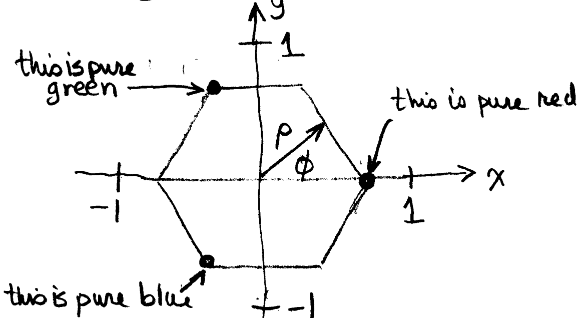
$\rho = \sqrt{x^2 + y^2}$ this is the saturation

$\phi = \text{ang}\,(x, y)$ this is the hue



There are two problems with this notation

1. saturation is not really independent of intensity (this is the important one)
2. fully saturated colors are on a hexagon and not a circle.



for $R=1, G=0, B=0$ $X = 2/\sqrt{6}$ $y = 0$

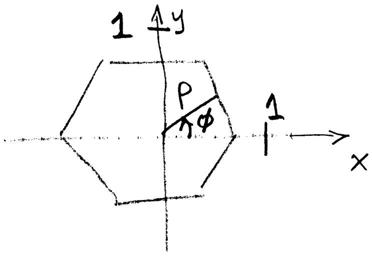
$R=0, G=1, B=0$ $X = -\frac{1}{\sqrt{6}}$ $y = \frac{1}{\sqrt{2}}$

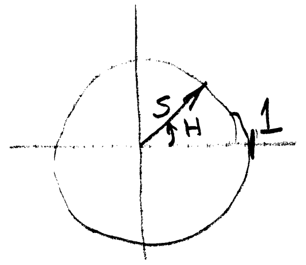
$R=0, G=0, B=1$ $X = -\frac{1}{\sqrt{6}}$ $y = -\frac{1}{\sqrt{2}}$

trick to make saturation independent of intensity
— divide by its maximum for that value of ϕ (hue).

You can get something like this

$$S = \frac{\rho}{\rho_{max}} = 1 - \frac{3\min(R,G,B)}{R+G+B} = 1 - \frac{\sqrt{3}}{I}\min(R,G,B)$$





You can do a similar thing for $\overset{\text{hue}}{\phi}$ to get a better mapping

$$\theta = \cos^{-1}\left[\frac{\frac{1}{2}(R-G)+(R-B)}{(R-G)^2+(R-B)(G-B)}\right]$$

and define $H = \begin{cases} \theta & G > B \\ 2\pi - \theta & G \leq B. \end{cases}$

21.3.4.2 HSI to RGB Conversion

The formulas vary depending upon which sector of the color circle you are in

for $0^\circ \leq H < 120^\circ$ $\quad R = \frac{I}{\sqrt{3}}\left[1 + \frac{S\cos(H)}{\cos(60^\circ - H)}\right]$ $\quad B = \frac{I}{\sqrt{3}}(1-S)$ $\quad G = \sqrt{3}I - R - B$

for $120^\circ \leq H < 240^\circ$ $\quad G = \frac{I}{\sqrt{3}}\left[1 + \frac{S\cos(H-120^\circ)}{\cos(180^\circ - H)}\right]$ $\quad R = \frac{I}{\sqrt{3}}(1-S)$ $\quad B = \sqrt{3}I - R - G$

for $240^\circ \leq H < 360^\circ$ $\quad B = \frac{I}{\sqrt{3}}\left[1 + \frac{S\cos(H-240^\circ)}{\cos(300^\circ - H)}\right]$ $\quad G = \frac{I}{\sqrt{3}}(1-S)$ $\quad R = \sqrt{3}I - G - B.$

21.3.5 Color Image Enhancement

1. Color Balance

objects in scene shifted in color due to differences in R,G,B channels.

1. check if gray objects are gray
2. check if highly saturated colors have correct hue
   (if blacks or whites occur at different gray levels you need to balance colors)
3. Solution – linear gray-scale transformation of R, G, and B individuals
   usually only need to transform two of the three images
   - find light gray & dark gray sections of image
   - compute mean gray level of both areas in R, G and B
   - use a linear stretch on two of images to make them match third

   (You can use a B/W step wedge to calibrate cameras).

2. Contrast and color enhancement

- typically you apply a normal image processing algorithm to the I (intensity) component of a HSI image.
- apply geometric transformations to a RGB image or a HSI image on a pixel by pixel basis

hue alteration – add a constant to the hue of each pixel
   a small angle change will "cool" (+) or "warm" (–) an image
   large changes are not as predictable – can be used to exaggerate color differences

saturation enhancement
   simply multiply the saturation of each pixel by a constant
   you can use use a non-linear point transform but
      keep zero at the origin, otherwise you will change the color balance

21.3.5.3 Color Image Restoration

response of the eye
- detail is more visible in intensity than color
  - blurring } mostly intensity
  - noise }

general color image procedure.

1. use a linear point transform (on RGB image) to be sure that image properly fits gray scale and is balanced
2. Convert to HSI format
3. use an appropriate filter (low pass or median) to remove noise. To preserve average gray level the DC value must not change)
4. Use an optimum filter or edge enhancement filter to sharpen edges and enhance detail.
5. transform all three components (saturation enhancement) to utilize all of gray scale.

   Not sure this is correct. ?
6. Convert to RGB for printing and display.

21.3.5.4 Pseudo color.

generate a color image from a B/W one by assigning a color to each gray level using some rule

use because eye can differentiate more colors than grays on a monitor

maybe only 40 out of 256

21.3.6 Color Image Analysis

often stain microscope images with red, green & blue fluorescent dyes to identify different components.

However, objects in these images are rarely separated. Each object is typically visible in all three images. This spreading of an object from one component to the other two is called <u>color spread</u>.

model slide as

$$\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{b}$$

$\underline{b}$: black level offset of digitizer.

$\underline{y}$: color vector recorded by digitizer

$\underline{x}$: 3x1 color vector
3 color channels

$\underline{\underline{C}}$: models color spread of objects in digitized image

$$\underline{x} = \underline{\underline{C}}^{-1}(\underline{y} - \underline{b})$$

$\underline{x}$: original color brightness.

Some times different exposure times are used, Model as

$$\underline{y} = \underline{\underline{E}}\,\underline{\underline{C}}\,\underline{x} + \underline{b}$$

where $\underline{E}$ is a diagonal matrix

$$\begin{bmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{bmatrix}$$

which just scales,

$$\underline{x} = \underline{\underline{C}}^{-1}\underline{\underline{E}}^{-1}[\underline{y} - \underline{b}]$$

three color theory

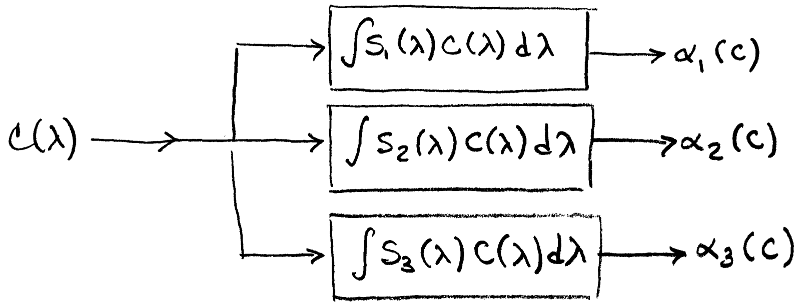
$$\alpha_i(c) = \int_{\lambda_{min}}^{\lambda_{max}} S_i(\lambda)\, C(\lambda)\, d\lambda \qquad i=1,2,3$$

spectral energy distribution of a "colored" light

response of different cones.

color sensation

classical three receptor model



Technically two colors that look identical could have different spectral distributions.

3.8 Color Matching and Reproduction

Consider three primary light sources such that any two cannot be combined to produce the third source, i.e. they are linearly independent.

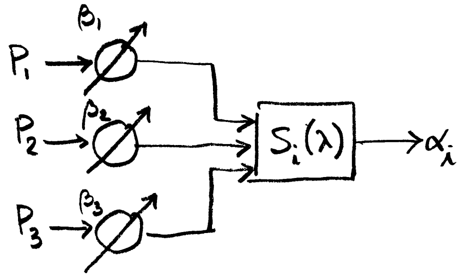
Let $\int P_k(\lambda)\,d\lambda = 1$

$$\boxed{\alpha_i(c) = \int \Big[\underbrace{\sum_{k=1}^{3} \beta_k P_k(\lambda)}_{\text{mix of three primary colors}}\Big] S_i(\lambda)\,d\lambda = \overbrace{\sum_{i=1}^{k} \beta_k \underbrace{\int S_i(\lambda) P_k(\lambda)\,d\lambda}_{\text{this is the } i\text{-th cone response generated by 1 unit of the } k\text{-th primary}}}^{\text{sum over each color}}} \quad \underbrace{i=1,2,3}_{\text{three sensors.}}$$

eye response to a source

define $\alpha_{i,k} \triangleq \alpha_i(P_k) = \int S_i(\lambda) P_k(\lambda)\,d\lambda$

$$\therefore \underbrace{\sum_{i=1}^{k} \beta_k \alpha_{i,k} = \alpha_i(c) = \int S_i(\lambda)\, c(\lambda)\,d\lambda \qquad i=1,2,3}_{\text{fundamental ``color matching'' equations}}$$



Given
- arbitrary color spectral distribution $C(\lambda)$
- primary sources $P_k(\lambda)$
- spectral sensitivity curves $S_i(\lambda)$

Solve for β_k $k=1,2,3$ to match $C(\lambda)$

In practice, calibrate sources against a reference <u>white</u> light source.

$$\text{Tri-stimulus values of color} \rightarrow T_k(c) = \frac{\beta_k}{w_k}, \quad k=1,2,3$$

β_k ← amount of each source in mix

w_k ← amount of each source required to match white

for white, $T_k(c) = 1$

$$\sum_{k=1}^{3} \underset{\uparrow}{w_k}\, \underset{\uparrow}{\alpha_{i,k}}\, \overset{\text{unknown component.}}{T_k(\lambda')} = \overset{\text{known response}}{S_i(\lambda')} \qquad i=1,2,3$$

"white" component

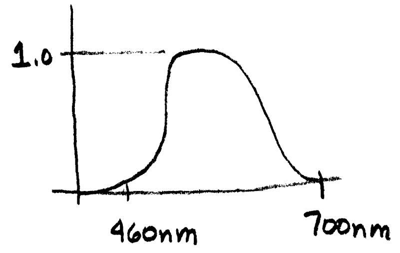
i-th cone response from k-th primary source

λ': one of standard three wavelengths of source

Laws of Color Matching

1. Any color can be matched by at most three colored lights.
2. The luminance of a color mixture is equal to the sum of the luminances of its components.

Luminance (or intensity) $Y = Y(c) = \int c(\lambda)\underset{\substack{\uparrow\\ \text{relative luminous efficiency.}}}{V(\lambda)}\,d\lambda$



Warning $c(\lambda) \neq \sum_{i=1}^{3} \underset{\substack{\uparrow\\ \text{white match}}}{w_k}\,\underset{\substack{\uparrow\\ \text{tri-stimulus.}}}{T_k}\,\underset{\substack{\uparrow\\ \text{distribution of source}}}{P_k(\lambda)}$

3. The human eye cannot resolve the components of a color mixture.
4. A color match at one luminance level holds over a wide range of luminances.
5. Color addition; If a color C_1 matches color C_2 and a color C_1' matches color C_2' then the mixture of C_1 and C_1' matches the mixture of C_2 and C_2'.
6. Color subtraction: If a mixture of C_1 and C_2 matches a mixture of C_1' and C_2' and if C_2 matches C_2' then C_1 matches C_1'
7. Transitive Law; If C_1 matches C_2 and if C_2 matches C_3, then C_1 matches C_3.
8. Color matches:
   a. $\alpha[C] = \alpha_1[C_1] + \alpha_2[C_2] + \alpha_3[C_3]$ direct match
   b. $\alpha[C] + \alpha_1[C_1] = \alpha_2[C_2] + \alpha_3[C_3]$
   c. $\alpha[C] + \alpha_1[C_1] + \alpha_2[C_2] = \alpha_3[C_3]$

   $\Big\}$ indirect matches.

Chromaticity Diagram

tristimulus coordinates

chromaticity coordinates $$t_k \triangleq \frac{T_k}{T_1 + T_2 + T_3}$$

because of the way this is defined ① $t_1 + t_2 + t_3 = 1$

② only two chromaticity coordinates are independent

$\Rightarrow$ color chromaticity projects 3-D color solid onto a plane.

t_1 & t_2 together represent hue and saturation

entire color space represented by (t_1, t_2, Y)

No practical set of primaries can create all colors. and are not physically realizable.

NTSC color systems.

receiver primary system (R_N, G_N, B_N)

three phosphor primaries that glow in R, G and B regions

color solid is a cube.

reference white different than for CIE system.

$$\begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix} = \begin{bmatrix} 1.910 & -0.533 & -0.288 \\ -0.985 & 2.000 & -0.028 \\ 0.058 & -0.118 & 0.896 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}.$$

transmission system (Y, I, Q)

Y = luminance (monochrome channel) of the color

high bandwidth ~ 2.5 MHz.

$\left.\begin{matrix} I \\ Q \end{matrix}\right\}$ quadrature components containing hue and saturation information

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -.274 & -.322 \\ 0.211 & -.523 & +.312 \end{bmatrix} \begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix}.$$

Many other systems.

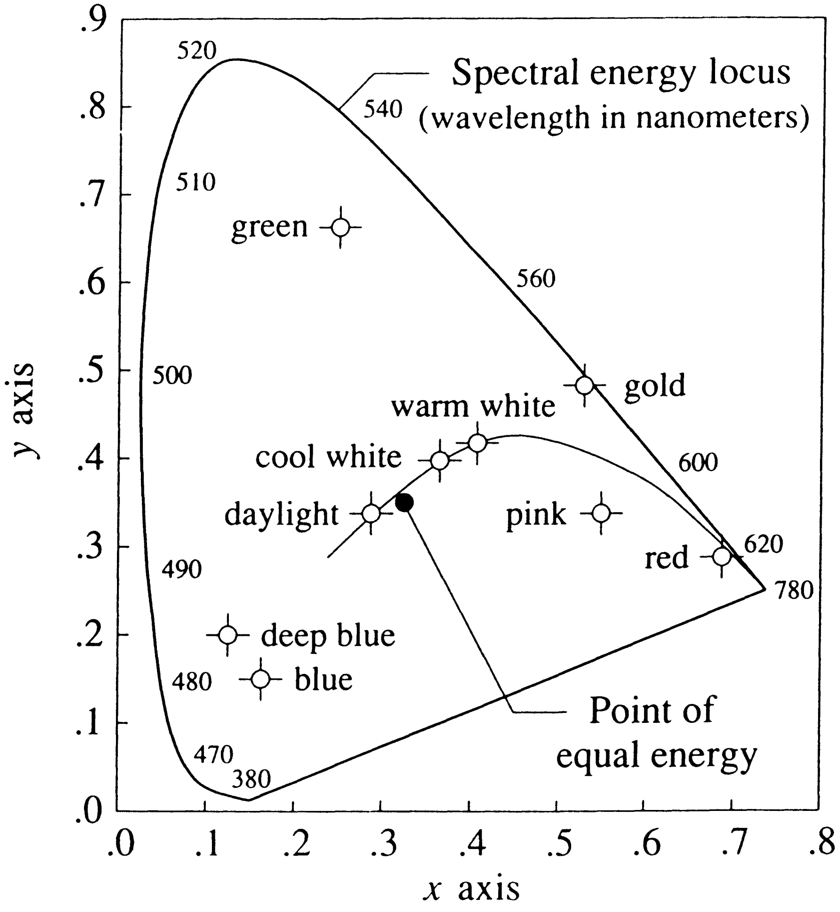


Figure 10.1: The diagram of CIE chromaticities. White light is at the center of the arch-shaped region. Fully saturated colors are along the outer edge of the diagram. The hue for a specific color is obtained by extending a line from white to the edge of the diagram passing through the color.

CIE = Commission Internationale de L'Eclairage

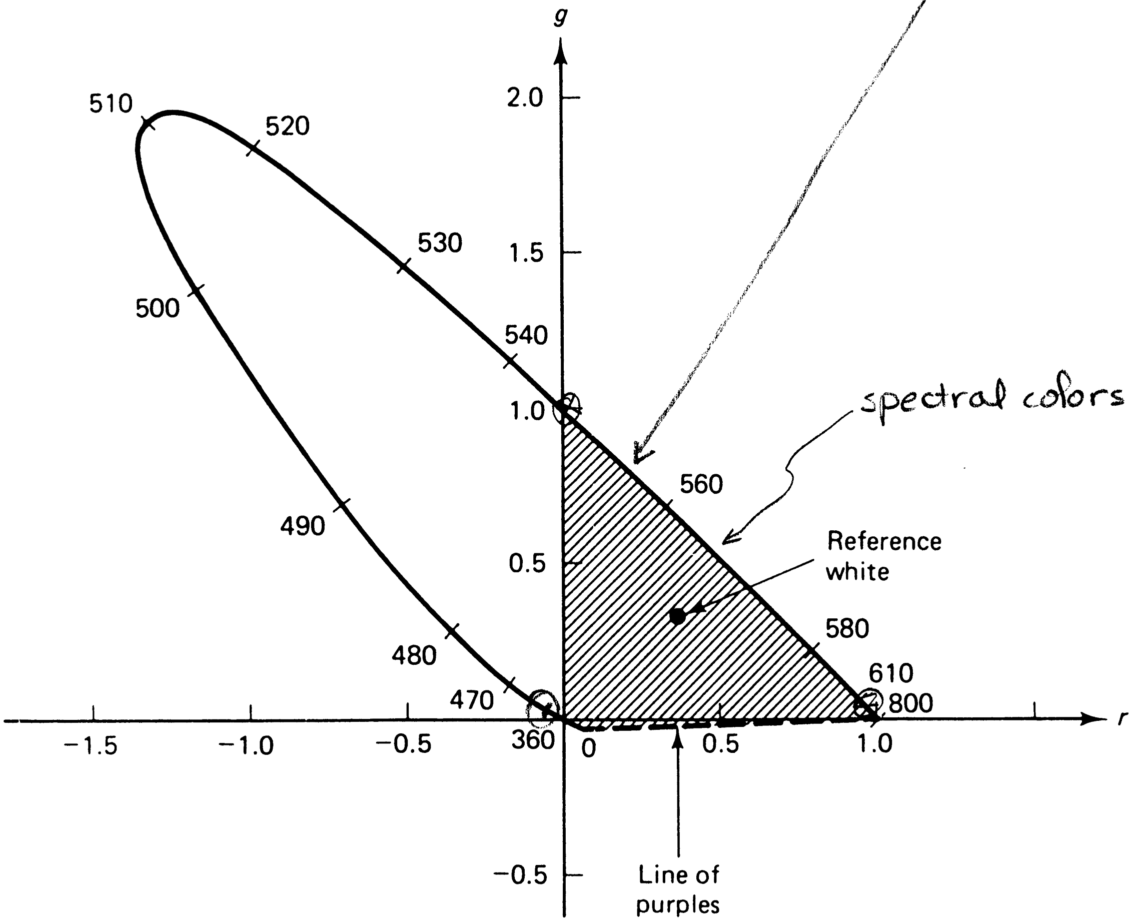
Primary sources (line sources)

λ_1 = 700 nm, red

λ_2 = 546.1 nm, green

λ_3 = 435.8 nm, blue.

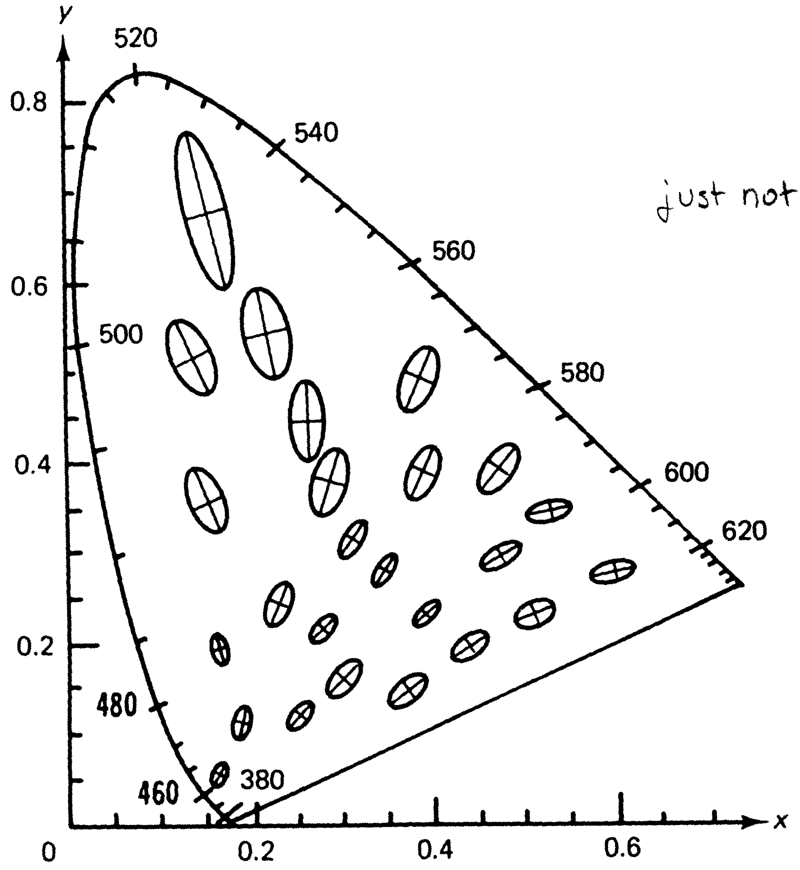
can only produce these colors



spectral colors

Figure 3.14 Chromaticity diagram for the CIE spectral primary system. Shaded area is the color gamut of this system.

1. contains all the visible colors
2. straight line joining blue (360nm) to red (780nm) contains the purple colors.
3. shaded region contains all colors reproducible by primary sources
4. reference white for CIE primary system has coordinates $(\frac{1}{3}, \frac{1}{3})$



just noticeably different (jnd) ellipses

Figure 3.15 Chromaticity diagram for the CIE XYZ color coordinate system. The (MacAdam) ellipses are the just noticeable color difference ellipses.

CIE primary system.
familiar RGB system.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Y = luminance.

There is a CIE uniform chromaticity scale which makes these differences uniform. Called CIE UCS.